Mathematics 433-A Fall 2001

Exam 2

November 13, 2001

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Name (10 *points*)

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

(15 points each) **Do any six** (6) **of the following.**

1. Given a group G, a subgroup H and any element $g \in G$, prove that the conjugate subgroup gHg^{-1} is isomorphic to H

2. Let G be a group acting on the set S. Let s be a fixed element in S and t an element in the orbit of s, say $t = as$. Prove the stabilizer of t in G is a conjugate subgroup of the stabilizer of s in G. Specifically, show $G_t = aG_sa^{-1}$.

3. Let $G = GL(2, \mathbf{R})$ act on the set $S = \mathbf{R}^2$ by left multiplication. That is, if $A \in GL(2, \mathbf{R})$ and $x \in \mathbf{R}^2$, the group action is defined by $(A, x) \mapsto Ax$. What is the stabilizer of $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$?

4. Given a group G, does the mapping $f : G \times G \to G$ given by $f(g, x) = xg^{-1}$ define a group action of G onto itself?

5. Use a group action to count the rotational symmetries of a tetrahedron. Be explicit about what you choose as your set S.

6. We say a group action of G on a set S is **faithful** if

$$(gs = s \; \forall s \in S) \Rightarrow (g = e).$$

   Let G be the dihedral group of symmetries of a square.

   (a) Is the action of G on the set of vertices a faithful action?

   (b) is the action of G on the set of diagonals a faithful action?

7. Let $K \subset H \subset G$ be subgroups of a **finite** group G. Prove the formula

$$[G : K] = [G : H] [H : K].$$

8. Let G be a group and $Aut(G)$ the group of automorphisms of $Aut(G)$. Prove or disprove: The set of inner automorphisms $Inn(G) = \{\phi \in Aut(G) : \phi(g) = xgx^{-1} \text{ for some } x \in G\}$ is a normal subgroup of G. Just determine normality, you may use, without proof, the fact that $Inn(G)$ is indeed a subgroup of $Aut(G)$.